Paper Reference(s)

6664/01

Edexcel GCE

Core Mathematics C2

Advanced Subsidiary

Thursday 26 May 2011 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C2), the paper reference (6664), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 9 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

1. $$f(x) = 2x^3 - 7x^2 - 5x + 4$$

(*a*) Find the remainder when f(x) is divided by (x – 1).

(2)

(*b*) Use the factor theorem to show that (x + 1) is a factor of f(x).

(2)

(*c*) Factorise f(x) completely.

(4)

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2. (*a*) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$(3 + bx)^5$$

where b is a non-zero constant. Give each term in its simplest form.

(4)

Given that, in this expansion, the coefficient of x^2 is twice the coefficient of x,

(*b*) find the value of b.

(2)

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3. Find, giving your answer to 3 significant figures where appropriate, the value of x for which

(*a*) $5^x = 10$,

(2)

(*b*) $\log_3 (x - 2) = -1$.

(2)

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4. The circle C has equation

$$x^2 + y^2 + 4x - 2y - 11 = 0.$$

Find

(*a*) the coordinates of the centre of C,

(2)

(*b*) the radius of C,

(2)

(*c*) the coordinates of the points where C crosses the y-axis, giving your answers as simplified surds.

(4)

5.

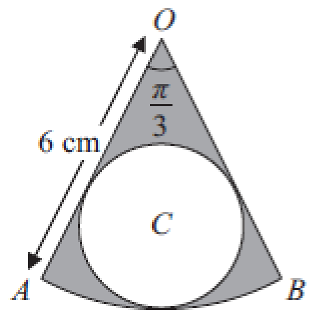

Figure 1

The shape shown in Figure 1 is a pattern for a pendant. It consists of a sector OAB of a circle centre O, of radius 6 cm, and angle $AOB = \frac{\pi}{3}$. The circle C, inside the sector, touches the two straight edges, OA and OB, and the arc AB as shown.

Find

(*a*) the area of the sector OAB,

(2)

(*b*) the radius of the circle C.

(3)

The region outside the circle C and inside the sector OAB is shown shaded in Figure 1.

(*c*) Find the area of the shaded region.

(2)

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6. The second and third terms of a geometric series are 192 and 144 respectively.

For this series, find

(*a*) the common ratio,

(2)

(*b*) the first term,

(2)

(*c*) the sum to infinity,

(2)

(*d*) the smallest value of n for which the sum of the first n terms of the series exceeds 1000.

(4)

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7. (*a*) Solve for $0 \leq x < 360°$, giving your answers in degrees to 1 decimal place,

$$3 \sin (x + 45°) = 2.$$

(4)

(*b*) Find, for $0 \leq x < 2\pi$, all the solutions of

$$2 \sin^2 x + 2 = 7\cos x,$$

giving your answers in radians.

You must show clearly how you obtained your answers.

(6)

8.

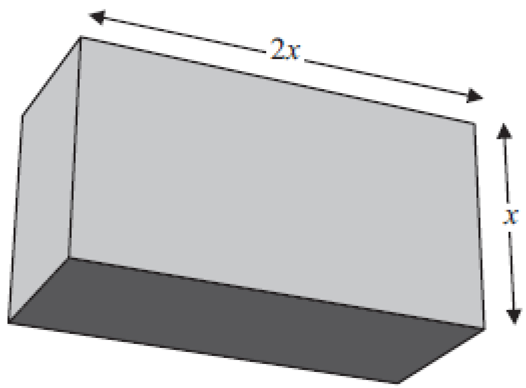

Figure 2

A cuboid has a rectangular cross-section where the length of the rectangle is equal to twice its width, x cm, as shown in Figure 2.

The volume of the cuboid is 81 cubic centimetres.

(*a*) Show that the total length, L cm, of the twelve edges of the cuboid is given by

$$L = 12x + \frac{162}{x^2}.$$

(3)

(*b*) Use calculus to find the minimum value of L.

(6)

(*c*) Justify, by further differentiation, that the value of L that you have found is a minimum.

(2)

9.

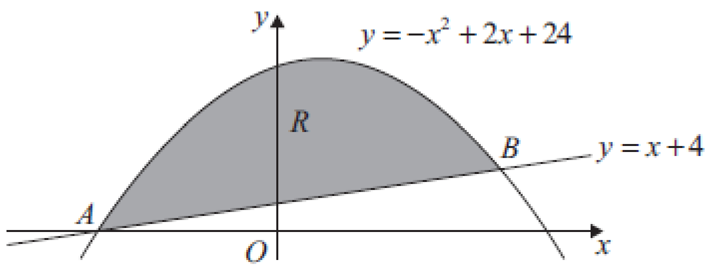

Figure 3

The straight line with equation $y = x + 4$ cuts the curve with equation $y = -x^2 + 2x + 24$ at the points A and B, as shown in Figure 3.

(a) Use algebra to find the coordinates of the points A and B.

(4)

The finite region R is bounded by the straight line and the curve and is shown shaded in Figure 3.

(b) Use calculus to find the exact area of R.

(7)

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme		Marks
1.	$\text{f}(x) = 2x^3 - 7x^2 - 5x + 4$		
(a)	Remainder $= \text{f}(1) = 2 - 7 - 5 + 4 = -6$	Attempts $\text{f}(1)$ or $\text{f}(-1)$.	M1
	$= -6$	-6	A1 **[2]**
(b)	$\text{f}(-1) = 2(-1)^3 - 7(-1)^2 - 5(-1) + 4$	Attempts $\text{f}(-1)$.	M1
	and so $(x+1)$ is a factor.	$\text{f}(-1) = 0$ with no sign or substitution errors **and for conclusion.**	A1 **[2]**
(c)	$\text{f}(x) = \{(x+1)\}(2x^2 - 9x + 4)$		M1 A1
	$= (x+1)(2x-1)(x-4)$		dM1 A1
			[4]
			8
2. (a)	$\{(3+bx)^5\} = (3)^5 + \underline{{}^5C_1(3)^4(bx)} + \underline{{}^5C_2(3)^3(bx)^2} + \ldots$	243 as a constant term seen.	B1
	$= 243 + 405bx + 270b^2x^2 + \ldots$	$405bx$	B1
		$({}^5C_1 \times \ldots \times x)$ or $({}^5C_2 \times \ldots \times x^2)$	M1
		$270b^2x^2$ or $270(bx)^2$	A1 **[4]**
(b)	$\{2(\text{coeff } x) = \text{coeff } x^2\} \Rightarrow 2(405b) = 270b^2$	Establishes an equation from their coefficients. Condone 2 on the wrong side of the equation.	M1
	So, $\left\{b = \frac{810}{270} \Rightarrow\right\} b = 3$	$b = 3$ (Ignore $b = 0$, if seen.)	A1
			[2]
			6
3. (a)	$5^x = 10$ and (b) $\log_3(x-2) = -1$		
	$x = \frac{\log 10}{\log 5}$ or $x = \log_5 10$		M1
	$x\,\{= 1.430676558\ldots\} = 1.43$ (3 sf)	1.43	A1 **cao**
			[2]
(b)	$(x-2) = 3^{-1}$	$(x-2) = 3^{-1}$ or $\frac{1}{3}$	M1 oe
	$x\,\{= \frac{1}{3} + 2\} = 2\frac{1}{3}$	$2\frac{1}{3}$ or $\frac{7}{3}$ or $2.\dot{3}$ or awrt 2.33	A1
			[2]
			4
4.	$x^2 + y^2 + 4x - 2y - 11 = 0$		
(a)	$\{\underline{(x+2)^2 - 4} + \underline{\underline{(y-1)^2}} - 1 - 11 = 0\}$	$(\pm 2, \pm 1)$, see notes.	M1
	Centre is $(-2, 1)$.	$(-2, 1)$.	A1 **cao** **[2]**
(b)	$(x+2)^2 + (y-1)^2 = 11 + 1 + 4$	$r = \sqrt{11 \pm \text{"1"} \pm \text{"4"}}$	M1
	So $r = \sqrt{11+1+4} \Rightarrow r = 4$	4 or $\sqrt{16}$ (Award A0 for ± 4).	A1 **[2]**
(c)	When $x = 0$, $y^2 - 2y - 11 = 0$	Putting $x = 0$ in C or their C.	M1
		$y^2 - 2y - 11 = 0$ or $(y-1)^2 = 12$, etc	A1 aef
	$y = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-11)}}{2(1)}$ $\left\{= \frac{2 \pm \sqrt{48}}{2}\right\}$	Attempt to use formula or a method of completing the square in order to find $y = \ldots$	M1
	So, $y = 1 \pm 2\sqrt{3}$	$1 \pm 2\sqrt{3}$	A1 **cao cso**
			[4]
			8

Question Number	Scheme		Marks
5. (a)	$\frac{1}{2}r^2\theta = \frac{1}{2}(6)^2\left(\frac{\pi}{3}\right) = 6\pi$ or 18.85 or awrt 18.8 (cm)^2	Using $\frac{1}{2}r^2\theta$ (See notes) 6π or 18.85 or awrt 18.8	M1 A1 **[2]**
(b)	$\sin\left(\frac{\pi}{6}\right) = \frac{r}{6-r}$ $\frac{1}{2} = \frac{r}{6-r}$ $6 - r = 2r \Rightarrow r = 2$	$\sin\left(\frac{\pi}{6}\right)$ or $\sin 30^{\circ} = \frac{r}{6-r}$ Replaces sin by numeric value $r = 2$	M1 dM1 A1 **cso** **[3]**
(c)	Area $= 6\pi - \pi(2)^2 = 2\pi$ or awrt 6.3 (cm)^2	their area of sector – πr^2 2π or awrt 6.3	M1 A1 **cao** **[2]** **7**
6. (a)	$\{ ar = 192$ and $ar^2 = 144 \}$ $r = \frac{144}{192}$ $r = \frac{3}{4}$ or 0.75	Attempt to eliminate a. $\frac{3}{4}$ or 0.75	M1 A1 **[2]**
(b)	$a(0.75) = 192$ $a\left\{= \frac{192}{0.75}\right\} = 256$	256	M1 A1 **[2]**
(c)	$S_\infty = \frac{256}{1-0.75}$ So, $\{S_\infty =\}$ 1024	Applies $\frac{a}{1-r}$ correctly using both their a **and** their $\lvert r\rvert < 1$. 1024	M1 A1 cao **[2]**
(d)	$\frac{256(1-(0.75)^n)}{1-0.75} > 1000$ $(0.75)^n < 1 - \frac{1000(0.25)}{256}$ $\left\{= \frac{6}{256}\right\}$ $n\log(0.75) < \log\left(\frac{6}{256}\right)$ $n > \frac{\log\left(\frac{6}{256}\right)}{\log(0.75)} = 13.0471042... \Rightarrow n = 14$	Applies S_n with their a and r and "uses" 1000 at any point in their working. (Allow with = or <). Attempt to isolate $+(r)^n$ from S_n formula. (Allow with = or >). Uses the power law of logarithms correctly. (Allow with = or >). $n = 14$	M1 M1 M1 A1 **cso** **[4]** **10**

Question Number	Scheme		Marks
7.	(a) $3\sin(x+45^\circ)=2$; $0\le x<360^\circ$ (b) $2\sin^2 x+2=7\cos x$; $0\le x<2\pi$		
(a)	$\sin(x+45^\circ)=\frac{2}{3}$, so $(x+45^\circ)=41.8103...$ $(\alpha=41.8103...)$	$\sin^{-1}\left(\frac{2}{3}\right)$ or awrt 41.8 or awrt 0.73^c	M1
	So, $x+45^\circ=\{138.1897...,\ 401.8103...\}$	$x+45^\circ$ = either "180 – their α" or "360° + their α" (α could be in radians).	M1
	and $x=\{93.1897...,\ 356.8103...\}$	Either awrt 93.2° or awrt 356.8°	A1
		Both awrt 93.2° and awrt 356.8°	A1
			[4]
(b)	$2(1-\cos^2 x)+2=7\cos x$	Applies $\sin^2 x=1-\cos^2 x$	M1
	$2\cos^2 x+7\cos x-4=0$	Correct 3 term, $2\cos^2 x+7\cos x-4\ \{=0\}$	A1 **oe**
	$(2\cos x-1)(\cos x+4)\ \{=0\}$, $\cos x=...$	Valid attempt at solving and $\cos x=...$	M1
	$\cos x=\frac{1}{2}$, $\{\cos x=-4\}$	$\cos x=\frac{1}{2}$	A1 cso
	$\left(\beta=\frac{\pi}{3}\right)$		
	$x=\frac{\pi}{3}$ or $1.04719...^c$	Either $\frac{\pi}{3}$ or awrt 1.05^c	B1
	$x=\frac{5\pi}{3}$ or $5.23598...^c$	Either $\frac{5\pi}{3}$ or awrt 5.24^c or 2π – their β	B1 ft
			[6] **10**
8. (a)	$\{V=\}$ $2x^2y=81$	$2x^2y=81$	B1 oe
	$\{L=2(2x+x+2x+x)+4y \Rightarrow L=12x+4y\}$		
	$y=\frac{81}{2x^2} \Rightarrow L=12x+4\left(\frac{81}{2x^2}\right)$	Making y the subject of their expression and substitute this into the correct L formula.	M1
	So, $L=12x+\frac{162}{x^2}$ **AG**	Correct solution only.	A1 **cso**
			[3]
(b)	$\frac{dL}{dx}=12-\frac{324}{x^3}$ $\{=12-324x^{-3}\}$	Either $12x\to12$ or $\frac{162}{x^2}\to\frac{\pm\lambda}{x^3}$	M1
		Correct differentiation (need not be simplified).	A1 aef
	$\left\{\frac{dL}{dx}=\right\}12-\frac{324}{x^3}=0 \Rightarrow x^3=\frac{324}{12};=27 \Rightarrow x=3$	$L'=0$ and "their $x^3=\pm$ value" or "their $x^{-3}=\pm$ value"	M1;
		$x=\sqrt[3]{27}$ or $x=3$	A1 **cso**
	$\{x=3,\}$ $L=12(3)+\frac{162}{3^2}=54$ (cm)	Substitute candidate's value of $x(\ne0)$ into a formula for L.	ddM1
		54	A1 **cao**
			[6]
(c)	$\{\text{For } x=3\}$, $\frac{d^2L}{dx^2}=\frac{972}{x^4}>0 \Rightarrow$ Minimum	Correct ft L'' and considering sign.	M1
		$\frac{972}{x^4}$ and >0 and conclusion.	A1 **[2]**
			11

Question Number	Scheme		Marks
9. (a)	Curve: $y = -x^2 + 2x + 24$, Line: $y = x + 4$		
	$\{\text{Curve} = \text{Line}\} \Rightarrow -x^2 + 2x + 24 = x + 4$	Eliminating y correctly.	B1
	$x^2 - x - 20\ \{= 0\} \Rightarrow (x-5)(x+4)\ \{= 0\} \Rightarrow x = ...$	Attempt to solve a ***resulting*** quadratic to give x = their values.	M1
	So, $x = 5, -4$	Both $x = 5$ and $x = -4$.	A1
	So corresponding y-values are $y = 9$ and $y = 0$.	.	B1ft **[4]**
(b)	$\left\{\int(-x^2 + 2x + 24)\,\mathrm{d}x\right\} = \underline{-\frac{x^3}{3} + \frac{2x^2}{2}} + 24x\ \{+c\}$	M1: $x^n \to x^{n+1}$ for any one term. 1st A1 at least two out of three terms. 2nd A1 for <u>correct answer</u>.	M1A1A1
	$\left[-\frac{x^3}{3} + \frac{2x^2}{2} + 24x\right]_{-4}^{5} = (......) - (......)$	Substitutes 5 and -4 (or their limits from part(a)) into an "integrated function" and subtracts, either way round.	dM1
	$\left\{\left(-\frac{125}{3} + 25 + 120\right) - \left(\frac{64}{3} + 16 - 96\right) = \left(103\frac{1}{3}\right) - \left(-58\frac{2}{3}\right) = 162\right\}$		
	Area of $\Delta = \frac{1}{2}(9)(9) = 40.5$	Uses correct method for finding area of triangle.	M1
		Area under curve – Area of triangle.	M1
	So area of R is $162 - 40.5 = 121.5$	121.5	A1 oe **cao**
			[7]
			11